64. ***Barn painting.*** Melanie can paint a certain barn by herself in x days. Her helper Melissa can paint the same barn by herself in $2x$ days. Write a rational expression for the fraction of the barn that they complete in one day by working together. Evaluate the expression for $x = 5$.

FIGURE FOR EXERCISE 64

GETTING MORE INVOLVED

65. ***Writing.*** Write a step-by-step procedure for adding rational expressions.

66. ***Writing.*** Explain why fractions must have the same denominator to be added. Use real-life examples.

7.5 COMPLEX FRACTIONS

In this section

- Complex Fractions
- Using the LCD to Simplify Complex Fractions
- Applications

In this section we will use the idea of least common denominator to simplify complex fractions. Also we will see how complex fractions can arise in applications.

Complex Fractions

A **complex fraction** is a fraction having rational expressions in the numerator, denominator, or both. Consider the following complex fraction:

$$\frac{\frac{1}{2}+\frac{2}{3}}{\frac{1}{4}-\frac{5}{8}} \quad \begin{array}{l} \leftarrow \text{Numerator of complex fraction} \\ \\ \leftarrow \text{Denominator of complex fraction} \end{array}$$

To simplify it, we can combine the fractions in the numerator as follows:

$$\frac{1}{2}+\frac{2}{3}=\frac{1\cdot 3}{2\cdot 3}+\frac{2\cdot 2}{3\cdot 2}=\frac{3}{6}+\frac{4}{6}=\frac{7}{6}$$

We can combine the fractions in the denominator as follows:

$$\frac{1}{4}-\frac{5}{8}=\frac{1\cdot 2}{4\cdot 2}-\frac{5}{8}=\frac{2}{8}-\frac{5}{8}=-\frac{3}{8}$$

Now divide the numerator by the denominator:

$$\frac{\frac{1}{2}+\frac{2}{3}}{\frac{1}{4}-\frac{5}{8}}=\frac{\frac{7}{6}}{-\frac{3}{8}}=\frac{7}{6}\div\left(-\frac{3}{8}\right)$$

$$=\frac{7}{6}\cdot\left(-\frac{8}{3}\right)$$

$$=-\frac{56}{18}$$

$$=-\frac{28}{9}$$

Using the LCD to Simplify Complex Fractions

A complex fraction can be simplified by writing the numerator and denominator as single fractions and then dividing, as we just did. However, there is a better method. The next example shows how to simplify a complex fraction by using the LCD of all of the single fractions in the complex fraction.

EXAMPLE 1

Using the LCD to simplify a complex fraction

Use the LCD to simplify

$$\frac{\frac{1}{2}+\frac{2}{3}}{\frac{1}{4}-\frac{5}{8}}.$$

Solution

The LCD of 2, 3, 4, and 8 is 24. Now multiply the numerator and denominator of the complex fraction by the LCD:

$$\frac{\frac{1}{2}+\frac{2}{3}}{\frac{1}{4}-\frac{5}{8}} = \frac{\left(\frac{1}{2}+\frac{2}{3}\right)24}{\left(\frac{1}{4}-\frac{5}{8}\right)24} \quad \text{Multiply the numerator and denominator by the LCD.}$$

$$= \frac{\frac{1}{2}\cdot 24+\frac{2}{3}\cdot 24}{\frac{1}{4}\cdot 24-\frac{5}{8}\cdot 24} \quad \text{Distributive property}$$

$$= \frac{12+16}{6-15} \quad \text{Simplify.}$$

$$= \frac{28}{-9}$$

$$= -\frac{28}{9}$$

calculator close-up

You can check Example 1 with a calculator as shown here.

```
(1/2+2/3)/(1/4-5
/8)▸Frac
            -28/9
```

CAUTION We simplify a complex fraction by multiplying the numerator and denominator of the *complex fraction* by the LCD. Do not multiply the numerator and denominator of each fraction in the complex fraction by the LCD.

In the next example we simplify a complex fraction involving variables.

EXAMPLE 2

A complex fraction with variables

Simplify

$$\frac{2-\frac{1}{x}}{\frac{1}{x^2}-\frac{1}{2}}.$$

helpful hint

When students see addition or subtraction in a complex fraction, they often convert all fractions to the same denominator. This is not wrong, but it is not necessary. Simply multiplying every fraction by the LCD eliminates the denominators of the original fractions.

Solution

The LCD of the denominators x, x^2, and 2 is $2x^2$:

$$\frac{2 - \frac{1}{x}}{\frac{1}{x^2} - \frac{1}{2}} = \frac{\left(2 - \frac{1}{x}\right)(2x^2)}{\left(\frac{1}{x^2} - \frac{1}{2}\right)(2x^2)} \qquad \text{Multiply the numerator and denominator by } 2x^2.$$

$$= \frac{2 \cdot 2x^2 - \frac{1}{x} \cdot 2x^2}{\frac{1}{x^2} \cdot 2x^2 - \frac{1}{2} \cdot 2x^2} \qquad \text{Distributive property}$$

$$= \frac{4x^2 - 2x}{2 - x^2} \qquad \text{Simplify.}$$

The numerator of this answer can be factored, but the rational expression cannot be reduced.

The general strategy for simplifying a complex fraction is stated as follows.

Strategy for Simplifying a Complex Fraction

1. Find the LCD for all the denominators in the complex fraction.
2. Multiply both the numerator and the denominator of the complex fraction by the LCD. Use the distributive property if necessary.
3. Combine like terms if possible.
4. Reduce to lowest terms when possible.

EXAMPLE 3

Simplifying a complex fraction

Simplify

$$\frac{\frac{1}{x-2} - \frac{2}{x+2}}{\frac{3}{2-x} + \frac{4}{x+2}}.$$

Solution

Because $x - 2$ and $2 - x$ are opposites, we can use $(x - 2)(x + 2)$ as the LCD. Multiply the numerator and denominator by $(x - 2)(x + 2)$:

$$\frac{\frac{1}{x-2} - \frac{2}{x+2}}{\frac{3}{2-x} + \frac{4}{x+2}} = \frac{\frac{1}{x-2}(x-2)(x+2) - \frac{2}{x+2}(x-2)(x+2)}{\frac{3}{2-x}(x-2)(x+2) + \frac{4}{x+2}(x-2)(x+2)}$$

$$= \frac{x + 2 - 2(x - 2)}{3(-1)(x + 2) + 4(x - 2)} \qquad \frac{x-2}{2-x} = -1$$

$$= \frac{x + 2 - 2x + 4}{-3x - 6 + 4x - 8} \qquad \text{Distributive property}$$

$$= \frac{-x + 6}{x - 14} \qquad \text{Combine like terms.}$$

study tip

Studying in an environment similar to the one in which you will be tested can increase your chances of recalling information. When possible, review for a test in the classroom in which you will take the test.

Applications

As their name suggests, complex fractions arise in some fairly complex situations.

EXAMPLE 4

Fast-food workers

A survey of college students found that $\frac{1}{2}$ of the female students had jobs and $\frac{2}{3}$ of the male students had jobs. It was also found that $\frac{1}{4}$ of the female students worked in fast-food restaurants and $\frac{1}{6}$ of the male students worked in fast-food restaurants. If equal numbers of male and female students were surveyed, then what fraction of the working students worked in fast-food restaurants?

Solution

Let x represent the number of males surveyed. The number of females surveyed is also x. The total number of students working in fast-food restaurants is

$$\frac{1}{4}x + \frac{1}{6}x.$$

The total number of working students in the survey is

$$\frac{1}{2}x + \frac{2}{3}x.$$

So the fraction of working students who work in fast-food restaurants is

$$\frac{\frac{1}{4}x + \frac{1}{6}x}{\frac{1}{2}x + \frac{2}{3}x}.$$

The LCD of the denominators 2, 3, 4, and 6 is 12. Multiply the numerator and denominator by 12 to eliminate the fractions as follows:

$$\begin{aligned}
\frac{\frac{1}{4}x + \frac{1}{6}x}{\frac{1}{2}x + \frac{2}{3}x} &= \frac{\left(\frac{1}{4}x + \frac{1}{6}x\right)12}{\left(\frac{1}{2}x + \frac{2}{3}x\right)12} && \text{Multiply numerator and denominator by 12.} \\
&= \frac{3x + 2x}{6x + 8x} && \text{Distributive property} \\
&= \frac{5x}{14x} && \text{Combine like terms.} \\
&= \frac{5}{14} && \text{Reduce.}
\end{aligned}$$

So $\frac{5}{14}$ (or about 36%) of the working students work in fast-food restaurants. ■

WARM-UPS

True or false? Explain your answer.

1. The LCD for the denominators 4, x, 6, and x^2 is $12x^3$.
2. The LCD for the denominators $a - b$, $2b - 2a$, and 6 is $6a - 6b$.
3. The fraction $\frac{4117}{7983}$ is a complex fraction.
4. The LCD for the denominators $a - 3$ and $3 - a$ is $a^2 - 9$.
5. The largest common denominator for the fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ is 24.

WARM-UPS

(continued)

Questions 6–10 refer to the following complex fractions:

a) $\dfrac{\frac{1}{2}+\frac{x}{3}}{\frac{1}{4}+\frac{1}{5}}$ **b)** $\dfrac{1+\frac{2}{b}}{\frac{2}{a}+5}$

c) $\dfrac{x-\frac{1}{2}}{x+\frac{3}{2}}$ **d)** $\dfrac{\frac{1}{2}+\frac{1}{3}}{1+\frac{1}{2}}$

6. To simplify (a), we multiply the numerator and denominator by $60x$.

7. To simplify (b), we multiply the numerator and denominator by $\frac{ab}{ab}$.

8. The complex fraction (c) is equivalent to $\frac{2x-1}{2x+3}$.

9. If $x \neq -\frac{3}{2}$, then (c) represents a real number.

10. The complex fraction (d) can be written as $\frac{5}{6} \div \frac{3}{2}$.

7.5 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What is a complex fraction?

2. What are the two ways to simplify a complex fraction?

Simplify each complex fraction. See Example 1.

3. $\dfrac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{4}-\frac{1}{2}}$

4. $\dfrac{\frac{1}{3}-\frac{1}{4}}{\frac{1}{3}+\frac{1}{6}}$

5. $\dfrac{\frac{2}{5}+\frac{5}{6}-\frac{1}{2}}{\frac{1}{2}-\frac{1}{3}+\frac{1}{15}}$

6. $\dfrac{\frac{2}{5}-\frac{2}{9}-\frac{1}{3}}{\frac{1}{3}+\frac{1}{5}+\frac{2}{15}}$

7. $\dfrac{3+\frac{1}{2}}{5-\frac{3}{4}}$

8. $\dfrac{1+\frac{1}{12}}{1-\frac{1}{12}}$

9. $\dfrac{1-\frac{1}{6}+\frac{2}{3}}{1+\frac{1}{15}-\frac{3}{10}}$

10. $\dfrac{3-\frac{2}{9}-\frac{1}{6}}{\frac{5}{18}-\frac{1}{3}-2}$

Simplify each complex fraction. See Example 2.

11. $\dfrac{\frac{1}{a}+\frac{3}{b}}{\frac{1}{b}-\frac{3}{a}}$

12. $\dfrac{\frac{1}{x}-\frac{3}{2}}{\frac{3}{4}+\frac{1}{x}}$

13. $\dfrac{5-\frac{3}{a}}{3+\frac{1}{a}}$

14. $\dfrac{4+\frac{3}{y}}{1-\frac{2}{y}}$

15. $\dfrac{\frac{1}{2}-\frac{2}{x}}{3-\frac{1}{x^2}}$

16. $\dfrac{\frac{2}{a}+\frac{5}{3}}{\frac{3}{a}-\frac{3}{a^2}}$

17. $\dfrac{\frac{3}{2b}+\frac{1}{b}}{\frac{3}{4}-\frac{1}{b^2}}$

18. $\dfrac{\frac{3}{2w}+\frac{4}{3w}}{\frac{1}{4w}-\frac{5}{9w}}$

Simplify each complex fraction. See Example 3.

19. $\dfrac{1-\frac{3}{y+1}}{3+\frac{1}{y+1}}$

20. $\dfrac{2-\frac{1}{a-3}}{3-\frac{1}{a-3}}$

21. $\dfrac{x + \dfrac{4}{x - 2}}{x - \dfrac{x + 1}{x - 2}}$

22. $\dfrac{x - \dfrac{x - 6}{x - 1}}{x - \dfrac{x + 15}{x - 1}}$

23. $\dfrac{\dfrac{1}{3 - x} - 5}{\dfrac{1}{x - 3} - 2}$

24. $\dfrac{\dfrac{2}{x - 5} - x}{\dfrac{3x}{5 - x} - 1}$

25. $\dfrac{1 - \dfrac{5}{a - 1}}{3 - \dfrac{2}{1 - a}}$

26. $\dfrac{\dfrac{1}{3} - \dfrac{2}{9 - x}}{\dfrac{1}{6} - \dfrac{1}{x - 9}}$

27. $\dfrac{\dfrac{1}{m - 3} - \dfrac{4}{m}}{\dfrac{3}{m - 3} + \dfrac{1}{m}}$

28. $\dfrac{\dfrac{1}{y + 3} - \dfrac{4}{y}}{\dfrac{1}{y} - \dfrac{2}{y + 3}}$

29. $\dfrac{\dfrac{2}{w - 1} - \dfrac{3}{w + 1}}{\dfrac{4}{w + 1} + \dfrac{5}{w - 1}}$

30. $\dfrac{\dfrac{1}{x + 2} - \dfrac{3}{x + 3}}{\dfrac{2}{x + 3} + \dfrac{3}{x + 2}}$

31. $\dfrac{\dfrac{1}{a - b} - \dfrac{1}{a + b}}{\dfrac{1}{b - a} + \dfrac{1}{b + a}}$

32. $\dfrac{\dfrac{1}{2 + x} - \dfrac{1}{2 - x}}{\dfrac{1}{x + 2} - \dfrac{1}{x - 2}}$

Simplify each complex fraction.

33. $\dfrac{\dfrac{2x - 9}{6}}{\dfrac{2x - 3}{9}}$

34. $\dfrac{\dfrac{a - 5}{12}}{\dfrac{a + 2}{15}}$

35. $\dfrac{\dfrac{2x - 4y}{xy^2}}{\dfrac{3x - 6y}{x^3y}}$

36. $\dfrac{\dfrac{ab + b^2}{4ab^5}}{\dfrac{a + b}{6a^2b^4}}$

37. $\dfrac{\dfrac{a^2 + 2a - 24}{a + 1}}{\dfrac{a^2 - a - 12}{(a + 1)^2}}$

38. $\dfrac{\dfrac{y^2 - 3y - 18}{y^2 - 4}}{\dfrac{y^2 + 5y + 6}{y - 2}}$

39. $\dfrac{\dfrac{x}{x + 1}}{\dfrac{1}{x^2 - 1} - \dfrac{1}{x - 1}}$

40. $\dfrac{\dfrac{a}{a^2 - b^2}}{\dfrac{1}{a + b} + \dfrac{1}{a - b}}$

Solve each problem. See Example 4.

41. ***Sophomore math.*** A survey of college sophomores showed that $\frac{5}{6}$ of the males were taking a mathematics class and $\frac{3}{4}$ of the females were taking a mathematics class. One-third of the males were enrolled in calculus, and $\frac{1}{5}$ of the females were enrolled in calculus. If just as many males as females were surveyed, then what fraction of the surveyed students taking mathematics were enrolled in calculus? Rework this problem assuming that the number of females in the survey was twice the number of males.

42. ***Commuting students.*** At a well-known university, $\frac{1}{4}$ of the undergraduate students commute, and $\frac{1}{3}$ of the graduate students commute. One-tenth of the undergraduate students drive more than 40 miles daily, and $\frac{1}{6}$ of the graduate students drive more than 40 miles daily. If there are twice as many undergraduate students as there are graduate students, then what fraction of the commuters drive more than 40 miles daily?

FIGURE FOR EXERCISE 42

GETTING MORE INVOLVED

43. ***Exploration.*** Simplify

$$\frac{1}{1+\frac{1}{2}},\quad \frac{1}{1+\frac{1}{1+\frac{1}{2}}},\quad \text{and}\quad \frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{2}}}}.$$

a) Are these fractions getting larger or smaller as the fractions become more complex?

b) Continuing the pattern, find the next two complex fractions and simplify them.

c) Now what can you say about the values of all five complex fractions?

44. ***Discussion.*** A complex fraction can be simplified by writing the numerator and denominator as single fractions and then dividing them or by multiplying the numerator and denominator by the LCD. Simplify the complex fraction

$$\frac{\frac{4}{xy^2}-\frac{6}{xy}}{\frac{2}{x^2}+\frac{4}{x^2y}}$$

by using each of these methods. Compare the number of steps used in each method, and determine which method requires fewer steps.

7.6 SOLVING EQUATIONS WITH RATIONAL EXPRESSIONS

In this section

- Equations with Rational Expressions
- Extraneous Solutions

Many problems in algebra can be solved by using equations involving rational expressions. In this section you will learn how to solve equations that involve rational expressions, and in Sections 7.7 and 7.8 you will solve problems using these equations.

Equations with Rational Expressions

We solved some equations involving fractions in Section 2.3. In that section the equations had only integers in the denominators. Our first step in solving those equations was to multiply by the LCD to eliminate all of the denominators.

EXAMPLE 1 **Integers in the denominators**

Solve $\frac{1}{2}-\frac{x-2}{3}=\frac{1}{6}$.

Solution

The LCD for 2, 3, and 6 is 6. Multiply each side of the equation by 6:

$$\frac{1}{2}-\frac{x-2}{3}=\frac{1}{6} \qquad \text{Original equation}$$

$$6\left(\frac{1}{2}-\frac{x-2}{3}\right)=6\cdot\frac{1}{6} \qquad \text{Multiply each side by 6.}$$

$$6\cdot\frac{1}{2}-\overset{2}{\cancel{6}}\cdot\frac{x-2}{\cancel{3}}=\cancel{6}\cdot\frac{1}{\cancel{6}} \qquad \text{Distributive property}$$

$$3-2(x-2)=1 \qquad \text{Simplify. Enclose } x-2 \text{ in parentheses.}$$

$$3-2x+4=1 \qquad \text{Distributive property}$$

$$-2x=-6 \qquad \text{Subtract 7 from each side.}$$

$$x=3 \qquad \text{Divide each side by } -2.$$

helpful hint

Note that it is not necessary to convert each fraction into an equivalent fraction with a common denominator here. Since we can multiply both sides of an equation by any expression we choose, we choose to multiply by the LCD. This tactic eliminates the fractions in one step.