Released Test Answer and Alignment Document Mathematics - Algebra 2

Performance Based Assessment

The following pages include the answer key for all machine-scored items, followed by the rubrics for the hand-scored items.

- The rubrics show sample student responses. Other valid methods for solving the problem can earn full credit unless a specific method is required by the item.
- In items where the scores are awarded for full and partial credit, the definition of partial credit will be confirmed during range-finding (reviewing sets of real student work).
- If students make a computation error, they can still earn points for reasoning or modeling.


| 10. | $a=2$ | N-RN.2 | Math 2 |
| :---: | :--- | :--- | :---: |
| 11. | Part A: see rubric <br> Part B: see rubric | HS.C.CCR | Math 3 |
| 12. | Part A: see rubric <br> Part B: see rubric | HS.D.3-6 | Math 3 |
| 13. | See rubric | HS.C.6.2 | Math 3 |
| 14. | Part A: see rubric <br> Part B: see rubric | HS.C.17.5 | Math 3 |
| 15. | Part A: see rubric <br> Part B: see rubric | HS.D.2-7 | Math 3 |
| 16. | Part A: see rubric <br> Part B: see rubric | HS.C.18.4 | Math 3 |
| 17. | Part A: see rubric <br> Part B: see rubric | HS.C.9.2 | Math 3 |
| 18. | Part A: see rubric <br> Part B: see rubric | Math 3 |  |



- This is machine-scored.

0 Student response is incorrect or irrelevant.

## \#11 Part B

| Score | Description |
| :---: | :---: |
| 3 | Student response includes the following 3 elements. <br> - Reasoning component $=3$ point <br> o The equation is correctly rewritten in factored form and the key feature is correctly explained. <br> o The equation is correctly rewritten by completing the square and the key feature is correctly explained. <br> o $\quad f(0)$ is correctly evaluated and the key feature is correctly explained. <br> Sample Student Response: <br> Student reasons that Angela has factored the expression in order to find the x intercepts of the function. $\begin{aligned} & f(x)=4 x^{2}+8 x-5 \\ & f(x)=(2 x-1)(2 x+5) \end{aligned}$ |


|  | If the function is set equal to 0 , then $\begin{aligned} & 0=(2 x-1)(2 x+5) \\ & 2 x-1=0 \text { and } 2 x+5=0 \\ & x=\frac{1}{2} \text { and } x=-\frac{5}{2} \end{aligned}$ <br> Therefore the $x$-intercepts are $\left(\frac{1}{2}, 0\right)$ and $\left(-\frac{5}{2}, 0\right)$. <br> Student reasons that Benjamin completes the square on the function in order to find the vertex. $\begin{aligned} & f(x)=4 x^{2}+8 x-5 \\ & f(x)=4\left(x^{2}+2 x\right)-5 \\ & f(x)=4\left(x^{2}+2 x+1\right)-5-4 \\ & f(x)=4(x+1)^{2}-9 \end{aligned}$ <br> Therefore the vertex is $(-1,-9)$. <br> Student reasons that Carla evaluates $f(0)$ to find the $y$-intercept at $(0,-5)$. <br> Note: The response does not need to find the value of the key feature in order to identify the key feature. |
| :---: | :---: |
| 2 | Student response includes 2 of the 3 elements. |
| 1 | Student response includes 1 of the 3 elements. |
| 0 | Student response is incorrect or irrelevant. |


| Score | Description |
| :---: | :---: |
| 4 | Student response includes the following 4 elements. <br> - Modeling component $=4$ points <br> o Model for Van: C $=8.9 p$ <br> o Model for Truck: $\mathrm{C}=6.7 \mathrm{p}+12$ <br> - C represents the total cost, and $p$ represents the fuel cost per gallon. <br> Justification for the model for the van <br> o Justification for the model for the truck <br> Sample Student Response: <br> Van: $C=8.9 p$ <br> Truck: $C=6.7 p+12$ <br> $C$ represents the total cost, and $p$ represents the fuel cost per gallon. <br> First determine the total distance that would be traveled by each vehicle type. <br> For the truck, use the sum of the segment lengths connecting a circuit of the given points, starting and ending at the distribution center. For the van, determine the distance from the distribution center to each point and multiply the sum by 2 (to give the round-trip distances). Divide these values by the respective fuel efficiency values to determine the total number of gallons of fuel used. Finally, write linear equations based on these values and the loading dock fee in the case of the truck. |
| 3 | Student response includes 3 of the 4 elements. |
| 2 | Student response includes 2 of the 4 elements. |
| 1 | Student response includes 1 of the 4 elements. |
| 0 | Student response is incorrect or irrelevant. |


| Score | Description |
| :---: | :---: |
| 2 | Student response includes the following 2 elements. <br> - Modeling component $=1$ point <br> o Justification for the computation <br> - Computation component $=1$ point <br> o Reporting the findings <br> Sample Student Response: <br> It is more cost efficient to use the van when this inequality is true: $\begin{aligned} & 6.7 p+12>8.9 p \\ & 12>2.2 p \\ & 5.45>p \end{aligned}$ <br> When the cost of fuel is less than $\$ 5.45$ per gallon, it is more cost efficient to drive the van. |
| 1 | Student response includes 1 of the 2 elements. |
| 0 | Student response is incorrect or irrelevant. |

## \#13 Rubric

## Score $\quad$ Description

3
Student response includes the following 3 elements.

- Reasoning component $=2$ points
o Correct explanation of a method to prove that there are no solutions to the system of inequalities
o Correct explanation of a change to the system that would result in there being a solution set
- Computation component $=1$ point
o Correct ordered pair from the solution set, based on the change described

Sample Student Response:
In order to prove that the system of inequalities has no solution, you could graph each inequality.

The graph of $y \geq x^{2}+3$ is a parabola that opens upwards with vertex $(0,3)$. The parabola is a solid line, and its interior is shaded.

The graph of $y<\frac{x}{2}+1$ is a dashed line that passes through the point $(0,1)$, and has slope $\frac{1}{2}$. The graph is shaded below the dashed line. Neither the boundary lines nor the shaded regions of the graphs intersect, so there are no solutions to the system.

In order to change the system so that there is a solution set, the second inequality could be changed to $y>\frac{x}{2}+1$. The shading would then be above the dashed line, and the solution set would be the parabola and all of the points in its interior. One point would be $(0,4)$.

Note:

- The shading or overlap of the solution region must be stated or implied to receive credit for a valid explanation. Graphing or intersection alone does not imply that the shaded areas of the inequalities overlap.

|  | • Other valid changes and valid points may be accepted. |
| :--- | :--- |
| $\mathbf{2}$ | Precision point deductions could include, but are not limited to: <br> imprecise mathematical language, such as labeling the inequalities <br> as functions or equations; or, incorrect description or drawing of <br> graphed boundary lines such as using a solid line when a dashed <br> line is required. |
| $\mathbf{1}$ | Student response includes 2 of the 3 elements. |
| $\mathbf{0}$ | Student response includes 1 of the 3 elements. |


| Score | Description |
| :---: | :---: |
| 2 | Student response includes the following 2 elements. <br> - Reasoning component $=1$ point <br> o Justification for the computation referencing the data in the table <br> - Computation component $=1$ point <br> o The correct answer is $\$ 2.52$. An answer in the range of $\$ 2.50$ to $\$ 2.52$ is acceptable. <br> Sample Student Response 1: <br> Find the weighted average of the weight of the books and multiply it by the weight-based fee: <br> There are 24 books sold. The total weight of the books is 336 ounces. <br> $336 \div 24=14 ; 14 \times \$ 0.18=\$ 2.52$. This is the weighted average of all the data by frequency. <br> Sample Student Response 2: <br> Six (6) is the mode of the books sold. Use the corresponding weight (14) and multiply that weight by the original fee ( $\$ 0.18$ ). $14 \times 0.18=\$ 2.52$. <br> Note: Responses that do not have a correct calculation but show partial understanding of reasoning can receive 1 point. |
| 1 | Student response includes 1 of the 2 elements. |
| 0 | Student response is incorrect or irrelevant. |
|  | \#14 Part B |
| Score | Description |
| 2 | Student response includes the following 2 elements. <br> - Reasoning component $=2$ points <br> o An appropriate decision <br> o Justification for the decision |


|  | Sample Student Response: <br> There are three possible decisions that can be justified: <br> 1. To return to the original pricing of $\$ 0.18$ per ounce; or <br> 2. To recalculate a flat-rate fee based on the combined data from paperback and hardcover books; or <br> 3. To suggest a dual-pricing plan, one for paperback and one for hardcover. <br> Justification for decision 1 must appeal to the variability of the data. Because the hardcover books have greater weights than the paperbacks, a flat-rate of $\$ 2.52$ no longer makes sense and could cause the company to lose money. It may be preferable to return to the per-ounce shipping cost. A per-ounce shipping cost might make more sense if a single shipment were a combination of paperbacks and hardcovers. <br> Justification for decision 2 may not justify remaining with $\$ 2.52$. A student may take the weighted average of all the books to arrive at a new pricing plan ( 375 ounces/ 23 books $=16.3$ oz per book. 16.3 x $\$ 0.18=\$ 2.93$. A student may also use a variety of other methods which may not arrive at $\$ 2.52$ as long as they consider both paperback and hardcover books in combination to arrive at a new pricing plan. The use of quartiles is not acceptable due to the way the data is presented in the prompt. <br> Justification for decision 3 must appeal to two different statistics, one for the paperbacks and one for the hardcover. Perhaps the flat-rate $\$ 2.52$ could still apply to a shipment of paperbacks, but a flat-rate of $\$ 4.35$ (based on the hardcover mean of 24.1 ) could be applied to a shipment of hardcovers. <br> Notes: <br> o For a decision to receive credit it has to be based on the data provided in Part B. <br> o Justification for the decision must include some calculations based on the manipulation of the data provided in Part B. |
| :---: | :---: |
| 1 | Student response includes 1 of the 2 elements. |
| 0 | Student response is incorrect or irrelevant. |


| Score | Description |
| :---: | :---: |
| 1 | Student response includes the following element. <br> - Computation component $=1$ point <br> o Machine scored <br> Sample Student Response: |
| 0 | Student response is incorrect or irrelevant. |


| Score | Description |
| :---: | :---: |
| 2 | Student response includes the following 2 elements. <br> - Modeling component $=2$ points <br> o Identifying the two points <br> o Interpreting and explaining the meaning of the two points <br> Sample Student Response: <br> The point $(2,2)$ satisfies the constraints, and $(3,3)$ does not satisfy the constraints. The point $(2,2)$ represents eating 2 hot dogs and 2 hamburgers. This satisfies both constraints, so it is a possibility for Enrique. The point $(3,3)$ represents eating 3 hot dogs and 3 hamburgers. This satisfies neither constraint, so it is not a possibility |


|  | for Enrique. He could eat 2 hamburgers and 2 hotdogs and still be <br> under 1200 calories and 60 grams of fat, but if he ate 3 of each he <br> would exceed his weekly calorie and fat intake. |
| :---: | :--- |
| Note: The information in Part B can be scored independently of Part A <br> if the response does not specifically reference the graph in Part A. <br> However, any response in Part B that does specifically use the graph <br> in Part A for support or justification must be correct based on the <br> graph to receive credit. If the student graphs the line incorrectly but <br> interprets the meaning in a way that is consistent with the incorrect <br> graph, this point can still be earned. <br> Note: There are a variety of points that would work as long as they are <br> justified. Non-integer coordinates are acceptable. |  |
| $\mathbf{1}$ | Student response includes 1 of the 2 elements. |


| Score | Description |
| :---: | :---: |
| 3 | Student response includes the following 3 elements. <br> - Modeling component $=2$ points <br> o Valid model for the new area <br> o Valid model for the difference in areas <br> - Computation component $=1$ point <br> o Correct original area <br> Sample Student Response: <br> The area of a circle is $A=\pi r^{2}$, so the radio tower could previously reach an area of $A_{1}=\pi(20)^{2}=400 \pi$ square miles. <br> The area of land that the radio tower can now reach depends on how much the radius increased. Let $x$ be the number of miles that the radius increased. The area of land that the tower's signals can now reach is $A_{2}=\pi(20+x)^{2}=\pi\left(x^{2}+40 x+400\right)$ square miles. <br> To find the difference in the areas, subtract the two areas: $\begin{aligned} & A_{2}-A_{1}=\pi\left(x^{2}+40 x+400\right)-400 \pi \\ & =\pi\left(x^{2}+40 x\right) \text { square miles } \end{aligned}$ <br> (or equivalent model) <br> Note: All three components may be presented together in a single equation or expression. The models do not need to be identified. |
| 2 | Student response includes 2 of the 3 elements. |
| 1 | Student response includes 1 of the 3 elements. |
| 0 | Student response is incorrect or irrelevant. |


| Score | Description |
| :---: | :--- |
| $\mathbf{3}$ | Student response includes the following 3 elements. <br> $\quad$Modeling component $=2$ points <br> $0 \quad$ Correct work |


|  | o Correct explanation <br> - Computation component $=1$ point <br> o Correct answer <br> Sample Student Response: <br> The area of land that the radio station is able to reach after the radius has been increased is represented by $A=\pi(20+x)^{2}$ square miles. <br> Since there are about 74 people per square mile, the radio station is able to reach a population, $P$, of about $\begin{aligned} & \mathrm{P}=74 \bullet \mathrm{~A} \\ & \mathrm{P}=74 \pi(20+x)^{2} \text { people } \end{aligned}$ <br> It is given that this is equal to 160,000 people, so we can substitute to solve for x . $\begin{aligned} & 160,000=74 \pi(20+x)^{2} \\ & \frac{160,000}{74 \pi}=\frac{74 \pi(20+x)^{2}}{74 \pi} \\ & 688.2375917=(20+x)^{2} \\ & \sqrt{688.2375917}=\sqrt{(20+x)^{2}} \\ & 26.23428276=20+x \\ & 6.23428276=x \end{aligned}$ <br> This means that the radius that the radio tower's broadcast signals can reach increased by approximately 6.23 miles. <br> Note: The acceptable range for the third element [correct answer] is 6.22-6.25 miles. |
| :---: | :---: |
| 2 | Student response includes 2 of the 3 elements. |
| 1 | Student response includes 1 of the 3 elements. |
| 0 | Student response is incorrect or irrelevant. |


| Score | Description |
| :---: | :---: |
| 2 | Student response includes the following 2 elements. <br> - Reasoning component $=2$ points <br> o Gives logical explanation <br> o Correctly rewrites algebraic expressions <br> Sample Student Response: <br> The expression on the left side of the equation is the difference of two squares. $\left(x^{2}+y^{2}\right)^{2}-\left(x^{2}-y^{2}\right)^{2}=\left(\left(x^{2}+y^{2}\right)+\left(x^{2}-y^{2}\right)\right)\left(\left(x^{2}+y^{2}\right)-\left(x^{2}-y^{2}\right)\right)$ <br> Simplify the right side of the equation above. $\begin{aligned} \left(\left(x^{2}+y^{2}\right)+\left(x^{2}-y^{2}\right)\right)\left(\left(x^{2}+y^{2}\right)-\left(x^{2}-y^{2}\right)\right) & =\left(2 x^{2}\right)\left(y^{2}-\left(-y^{2}\right)\right. \\ & =\left(2 x^{2}\right)\left(2 y^{2}\right) \\ & =2^{2} x^{2} y^{2} \\ & =(2 x y)^{2} \end{aligned}$ <br> Now the expression $(2 x y)^{2}$ is exactly the right side of the given equation. Therefore, the given equation is true for all $x$ and $y$. <br> Notes: <br> o An explanation based on substituting one number for $x$ and $a$ different number for $y$ that correctly demonstrates the two sides of the equation are equal can receive credit for the explanation. $x$ and $y$ must both be positive integers but $x$ can be greater than or less than y in Part A. <br> o An explanation or work based on substituting numbers for variables can NOT receive credit for rewriting the algebraic expressions. |
| 1 | Student response includes 1 of the 2 elements. |
| 0 | Student response is incorrect or irrelevant. |


| Score | Description |
| :---: | :---: |
| 2 | Student response includes the following 2 elements. <br> - Reasoning component $=2$ points <br> o Explains that Jenna is correct <br> o Explains that $x>y$ when finding side lengths of right triangles <br> Sample Student Response: <br> The Pythagorean theorem states that for a right triangle, the sum of the squares of the leg lengths is equal to the square of the length of the hypotenuse, or $a^{2}+b^{2}=c^{2}$. Part A showed that <br> $\left(x^{2}+y^{2}\right)^{2}-\left(x^{2}-y^{2}\right)^{2}=(2 x y)^{2}$ for all $x$ and $y$. Therefore, <br> $\left(x^{2}-y^{2}\right)^{2}+(2 x y)^{2}=\left(x^{2}+y^{2}\right)^{2}$ is also true, and <br> $a=x^{2}-y^{2}, b=2 x y$, and $c=x^{2}+y^{2}$. <br> When using $x$ and $y$ to find side lengths of right triangles, $x$ must be greater than $y$. If $y$ were greater than $x$, then the term $x^{2}-y^{2}$ would result in a negative side length. <br> Note: <br> o The response must state or demonstrate that the three expressions listed ( $x^{2}+y^{2}, x^{2}-y^{2}$, and $2 x y$ ) can be substituted for $c, a$, and $b$ in the Pythagorean theorem. The hypotenuse (c) must correspond with $x^{2}+y^{2}$; the other two are interchangeable. |
| 1 | Student response includes 1 of the 2 elements. |
| 0 | Student response is incorrect or irrelevant. |


| Score | Description |
| :---: | :---: |
| 1 | Student response includes the following element. <br> - Reasoning component $=1$ point <br> o Valid explanation of a characteristic of an even function shown in the graph <br> Sample Student Response: <br> If $P$ is an even function, then $P(-x)$ would equal $P(x)$ for all values of $x$ in the domain of $P$. Graphically, this means that the graph is symmetric about the y-axis. The graph given seems to show this. |
| 0 | Student response is incorrect or irrelevant. |
|  | \#18 Part B |
| Score | Description |
| 3 | Student response includes the following 3 elements. <br> - Reasoning component $=3$ points <br> o Correct algebraic definition of an even function <br> o Correct application of definition <br> o Correct work showing that the function is even <br> Sample Student Response: <br> If $P(-x)=P(x)$, then $P$ is an even function. $\begin{aligned} P(-x) & =(-x)^{4}-8(-x)^{2}+16 \\ & =x^{4}-8 x^{2}+16 \\ & =P(x) \end{aligned}$ <br> Scoring note: The second scoring element, applying the definition correctly, will likely be shown in the first line of the student's work, where the student writes $P(-x)=(-x)^{4}-8(-x)^{2}+16$. |
| 2 | Student response includes 2 of the 3 elements. |
| 1 | Student response includes 1 of the 3 elements. |
| 0 | Student response is incorrect or irrelevant. |

