Math Analysis I Honors – Review for Chapter 4 Test 2014

1. Find the <u>domain</u> and <u>zeros</u> of the following:

   a) $f(x) = \frac{2x+1}{x^2-4x-12} = \frac{2x+1}{(x-6)(x+2)}$ $\quad x \neq 6, -2$ $\quad D: (-\infty, -2) \cup (-2, 6) \cup (6, \infty)$ $\quad Z: (-1/2, 0)$

   b) $f(x) = \sqrt{x-7} - 4$ $\quad \sqrt{x-7} \geq 0$, $x \geq 7$ $\quad D: [7, \infty)$ $\quad Z: (23, 0)$ $\rightarrow$ Zeros $\sqrt{x-7} - 4 = 0$

   $(\sqrt{x-7})^2 = (4)^2$

   $x - 7 = 16$

   $x = 23$

   c) $f(x) = x^2 + 2x + 1$ $\quad (x+1)^2 = 0$, $x = -1$ $\quad D: (-\infty, \infty)$ $\quad Z: (-1, 0) \leftarrow$ Dbl. Root

   d) $f(x) = \frac{\sqrt{6-x}}{x+3}$ $\quad x \neq -3$ $\quad \sqrt{6-x} \geq 0$, $6 - x \geq 0$, $6 \geq x$, $x \leq 6$ $\quad D: (-\infty, -3) \cup (-3, 6]$ $\quad Z: (6, 0)$

2. Let $f(x) = x^2 + 2x$ and $g(x) = x + 2$ and $h(x) = \sqrt{x+3}$. Find the following:

   a) $(f+g)(x) = x^2 + 3x + 2$

   b) $(f-g)(x) = x^2 + x - 2$

   c) $(fg)(x) = x^3 + 4x^2 + 4x$

   d) $(f/g)(x) = \frac{x^2+2x}{x+2} = \frac{x(\cancel{x+2})}{\cancel{x+2}} = x \;;\; x \neq -2$ $\quad x \neq -2 \rightarrow$

   e) $(f \circ g)(x) = (x+2)^2 + 2(x+2) = x^2 + 6x + 8$

   $x^2 + 4x + 4 + 2x + 4$

   f) $(g \circ f)(x) = x^2 + 2x + 2$

   g) $(h \circ g)(x) = \sqrt{(x+2)+3} = \sqrt{x+5}$

   h) $(f \circ h)(x) = (\sqrt{x+3})^2 + 2\sqrt{x+3} = x + 3 + 2\sqrt{x+3}$

   i) $f(h(6)) = f(3) = 3^2 + 2(3) = 9 + 6 = 15$

   $h(6) = \sqrt{6+3} = \sqrt{9} = 3$

   j) $(h \circ g \circ f)(1) = h(g(f(1))) = h(g(3)) = h(5) = \sqrt{8} = 2\sqrt{2}$

   $f(1) = 1^2 + 2(1) = 3$ $\quad g(3) = 3 + 2 = 5$

   k) $f(f^{-1}(5)) = 5$

   Inverse functions undo original to yield the inside.

   l) $h^{-1}(h(17)) = 17$

   m) $(f \circ f)(x) = (x^2+2x)^2 + 2(x^2+2x)$

   $x^4 + 4x^3 + 4x^2 + 2x^2 + 4x = x^4 + 4x^3 + 6x^2 + 4x$

   n) $(g(f(x-3))) = g(x^2 - 4x + 3) = x^2 - 4x + 5$

   $f(x-3) = (x-3)^2 + 2(x-3) = x^2 - 6x + 9 + 2x - 6 = x^2 - 4x + 3$

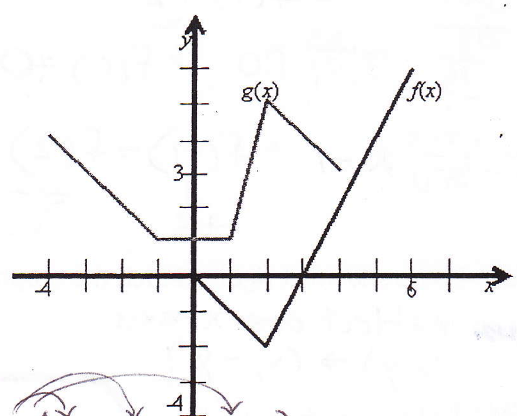
3. Given the graphs on the right for g(x) and f(x), find the indicated function values.

   a. $(f \circ g)(-3) = f(g(-3)) = f(3) = 0$

   b. $g(f(2)) = g(-2) = 2$

   c. $(f \circ g)(0) = f(g(0)) = f(1) = -1$

   d. $(g \circ f)(3) = g(f(3)) = g(0) = 1$



4. Given $f(x) = 3x^2 - 5x + 4$, find the difference quotients

   a) $\frac{f(x+h)-f(x)}{h}$ $\quad h \neq 0$

   $$= \frac{(3(x+h)^2 - 5(x+h) + 4) - (3x^2 - 5x + 4)}{h} = \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{5x} - 5h + \cancel{4} \; \cancel{-3x^2} + \cancel{5x} - \cancel{4}}{h}$$

   $$= \frac{6xh + 3h^2 - 5h}{h} = 6x + 3h - 5 \quad h \neq 0$$

   b) $\frac{f(3+h)-f(3)}{h}$ $\quad h \neq 0$

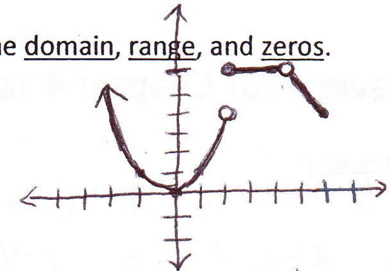
   $$= \frac{(3(3+h)^2 - 5(3+h) + 4) - (3(3)^2 - 5(3) + 4)}{h} = \frac{27 + 18h + 3h^2 - 15 - 5h + 4 - 16}{h}$$

   $(27 - 15 + 4)$

   $$= \frac{3h^2 + 13h}{h} = 3h + 13 \quad h \neq 0$$

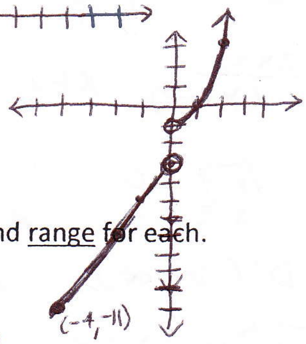
5. Graph the piecewise functions. Then find the domain, range, and zeros.

a) $g(x) = \begin{cases} x^2, & \text{if } x < 2 \\ 6, & \text{if } 2 \le x < 4 \\ 10 - x, & \text{if } 4 < x \le 6 \end{cases}$



D: $(-\infty, 4) \cup (4, 6]$
R: $[0, \infty)$ Zeros $(0,0)$

b) $f(x) = \begin{cases} x^2 - 1, & x > 0 \\ 2x - 3, & -4 \le x < 0 \end{cases}$



D: $[-4, 0) \cup (0, \infty)$
R: $[-11, -3) \cup (-1, \infty)$
Zeros $(1, 0)$

6. Find the inverses of the following functions. Give the domain and range for each.

D: $(-\infty, \infty)$ a) $f(x) = 3x - 7$
R: $(-\infty, \infty)$

$x = 3y - 7$
$y = \dfrac{x+7}{3} = f^{-1}(x)$

D: $(-\infty, \infty)$
R: $(-\infty, \infty)$

b) $f(x) = \sqrt{x+5} - 6$

D: $[-5, \infty)$
R: $[-6, \infty)$

$x = \sqrt{y+5} - 6$
$(\sqrt{y+5})^2 = (x+6)^2$
$f^{-1}(x) = y = (x+6)^2 - 5 = x^2 + 12x + 31$

D: $[-6, \infty)$ ← half of parabola
R: $[-5, \infty)$

c) $f(x) = \dfrac{4+x}{6-2x}$

D: $(-\infty, 3) \cup (3, \infty)$ Y

$x = \dfrac{4+y}{6-2y}$
$x(6-2y) = 4 + y$
$6x - 2xy = 4 + y$
$2xy + y = 6x - 4$
$y(2x+1) = 6x - 4$

$f^{-1}(x) = \dfrac{6x-4}{2x+1}$

D: $(-\infty, -1/2) \cup (-1/2, \infty)$
R: $(-\infty, 3) \cup (3, \infty)$ ← Domain of original

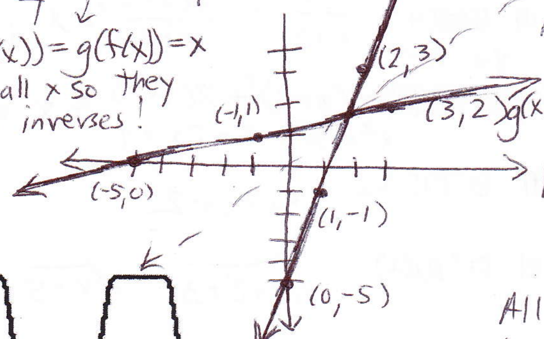
7. Show that the functions f(x) and g(x) are inverses using the Property of Inverse Functions. Then graph the inverse functions on the same set of axes.

$f(x) = 4x - 5$ $\qquad g(x) = \dfrac{x+5}{4}$

$g(f(x)) = \dfrac{(4x-5)+5}{4} = \dfrac{4x}{4} = x$ ✓

$f(g(x)) = 4\left(\dfrac{x+5}{4}\right) - 5 = x + 5 - 5 = x$ ✓

→ $f(g(x)) = g(f(x)) = x$ for all x so they are inverses!



Reflected over line y=x
All pts. (x,y) become (y,x)

8. Given the below periodic graph of f(x), find:

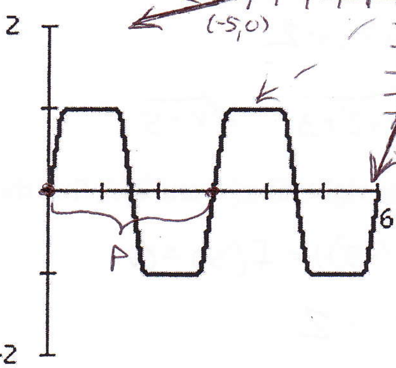
a) the fundamental period = 3 ← distance on x-axis it takes to repeat

b) the amplitude $= \dfrac{\max y - \min y}{2} = \dfrac{|1 - -1|}{2} = \dfrac{2}{2} = 1$

c) f(1000) $\quad 3\overline{)1000}$ = 333 R1 $= f(1) = 1$

d) f(99) $\quad 3\overline{)99}$ = 33 R0 $= f(0) = 0$

e) f(-1000) $\quad 3\overline{)-1000}$ = -333 R -1 $= f(-1) = f(2) = -1$
+3

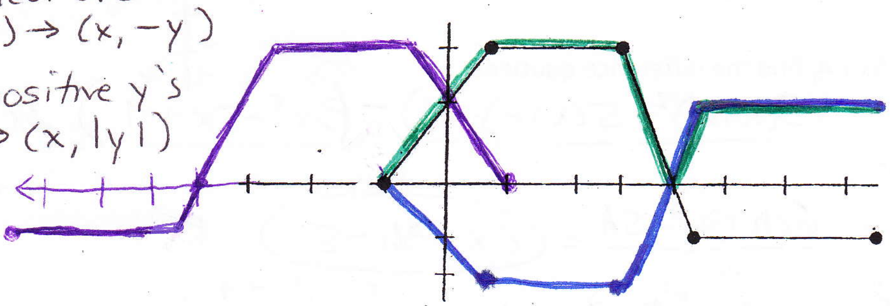


9. The graph of y=f(x) is shown at right. Sketch the graph of each of the following equations. Be accurate.

Y= $-f(x)$ reflect ove x-axis $(x,y) \to (x,-y)$

Y= $|f(x)|$ All positive y's $(x,y) \to (x,|y|)$

Y= $f(-x)$ Reflect over y-axis $(x,y) \to (-x,y)$



10. $f(-x) = 2(-x) - 10 = -2x - 10$ Neither

b) $f(-x) = 5(-x) - 2(-x)^5 = -5x + 2x^5 = -f(x)$ Odd

c) $f(-x) = 3(-x)^6 - 4(-x)^2 + 1 = 3x^6 - 4x^2 + 1 = f(x)$ Even