How to Approach a Linear Programming Problem

A contractor builds two types of homes. The Carolina requires one lot, $160,000 capital,

and 160 worker-days of labor, whereas the Savannah requires one lot, $240,000 capital,

and 160 worker-days of labor. The contractor owns 300 lots and has $48,000,000 available

capital and 43,200 worker-days labor. The profit on the Carolina is $40,000 and the

profit on the Savannah is $50,000. List the corner points of the feasible region

and find how many of each type of home should be built to maximize profit.

Find the maximum profit.

If you hate word problems, this one should be enough to send shivers up your spine. Don't let this intimidate you - we're going to handle this on a step by step basis.

First - look at what the problem wants us to determine

The question is, "...find how many of each type of home should be built to maximize profit."

Specifically, this means determine how many Carolina homes and how many Savannah homes they should build to maximize profit. So now we can name our variables. We'll let

x = the number of Carolina homes to be built

y = the number of Savannah homes to be built

Eventually, we're going to solve for x and y.

Second - develop the system of inequalities based on the limitations

Notice that each house requires a lot, money and labor. However, the contractor only has limited amounts of each of these. So as the contractor decides how many houses to be built, they have to keep in mind these limitations. We'll deal with these now.

**Lot Limitation**

Each Carolina home requires one lot. We're going to build "x" Carolina homes (remember - x represents the number of Carolina homes). So the number of lots we'll need for our Carolina homes can be represented by the expression: **1x**.

Each Savannah home also requires one lot. We're going to build "y" Savannah homes. So the number of lots we'll need for our Savannah homes can be represented by the expression: **1y**.

The total number of lots we'll need for both the Carolina and Savannah homes can be represented by the expression: **1x + 1y** (or **x + y)**.

The contractor only has 300 lots, so we can now express our first inequality:

http://vc.ws.edu/math1022/week7/linear1.gif

**Money Limitation**

Each Carolina home requires $160,000 capital. If the contractor is going to build "x" Carolina homes, the total capital they will need can be expressed by: **160000x**.

Each Savannah home requires $240,000 capital. If the contractor is going to build "y" Savannah homes, the total capital they will need can be expressed by: **240000y**.

The total amount of capital the contractor will need for all these homes is therefore: **160000x + 240000y**.

The contractor has $48,000,000 of total capital, so we can now express our second inequality:

http://vc.ws.edu/math1022/week7/linear2.gif

**Worker-day Limitation**

Each Carolina home requires160 worker-days of labor. If the contractor is going to build "x" Carolina homes, the total worker-days they will need can be expressed by: **160x**.

Each Savannah home requires 160 worker-days of labor. If the contractor is going to build "y" Savannah homes, the total worker-days they will need can be expressed by: **160y**.

The total amount of worker-days the contractor will need for all these homes is therefore: **160x + 160y**.

The contractor has 43,200 of total worker-days of labor, so we can now express our third inequality:

http://vc.ws.edu/math1022/week7/linear3.gif

**Reality Check**

We've taken care of our three limitations, but we still have one more item to take care. I call it our Reality Check.

Recall that x = the number of Carolina homes to be built. There's a minimum number of homes we can build. That minimum number is 0. Therefore

http://vc.ws.edu/math1022/week7/linear4.gif

We can use similar rationale for the Savannah homes. So we write:

http://vc.ws.edu/math1022/week7/linear5.gif

We now have our system of inequalities

http://vc.ws.edu/math1022/week7/linear1.gif

http://vc.ws.edu/math1022/week7/linear2.gif

http://vc.ws.edu/math1022/week7/linear3.gif

http://vc.ws.edu/math1022/week7/linear4.gifhttp://vc.ws.edu/math1022/week7/linear5.gif

**We want to maximize Profit**

Each Carolina home earns $40,000 profit. If the contractor is going to build "x" Carolina homes, the total profit they will get can be expressed by: **40000x**.

Each Savannah home earns $50000. If the contractor is going to build "y" Savannah homes, the total profit they will get can be expressed by: **50000y**.

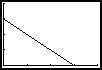
The total amount of profit the contractor will receive for all these homes is therefore: **40000x + 50000y**.

We want to maximize this expression.

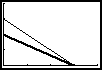
Third - graph this system of inequalities

Start with the two inequalities http://vc.ws.edu/math1022/week7/linear4.gifand http://vc.ws.edu/math1022/week7/linear5.gif. These two inequalities restrict our graph to Quadrant I. So when you draw your axes on your paper, just draw the first quadrant.

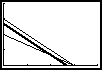
Now we'll graph http://vc.ws.edu/math1022/week7/linear1.gif. Determine two points (set x = 0 and solve for y, then set y = 0 and solve for x). Your line should look like this:

The x intercept is (300,0) and the y intercept is (0,300). Remember that we are **shading below** this line.

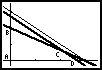
Next, we'll graph http://vc.ws.edu/math1022/week7/linear2.gif. Again, determine two points (set x = 0 solve for y, set y = 0 and solve for x).

Again, we're going to **shade below** this line.

Next, we'll graph http://vc.ws.edu/math1022/week7/linear3.gif. Again determine the two points.

. Again, we're going to **shade below**.

So our feasible region looks as below. (Since I can't figure out how to get the calculator to shade the way I want it, the shading should stop at the x and y axes and only go up to the bottom darkened lines).

I have labeled the four corner points of our shaded region.

Fourth - Determine the coordinates of the corner points

Each corner point is formed by the intersection of two lines from our system of inequalities.

For instance, corner point A uses the two inequalities http://vc.ws.edu/math1022/week7/linear4.gifand http://vc.ws.edu/math1022/week7/linear5.gif. Corner point A is the intersection of the two lines http://vc.ws.edu/math1022/week7/linear11.gifand http://vc.ws.edu/math1022/week7/linear12.gif.

Corner point A is the solution to the system of equations http://vc.ws.edu/math1022/week7/linear11.gif, http://vc.ws.edu/math1022/week7/linear12.gif! So the coordinates of point A is **(0,0)**.

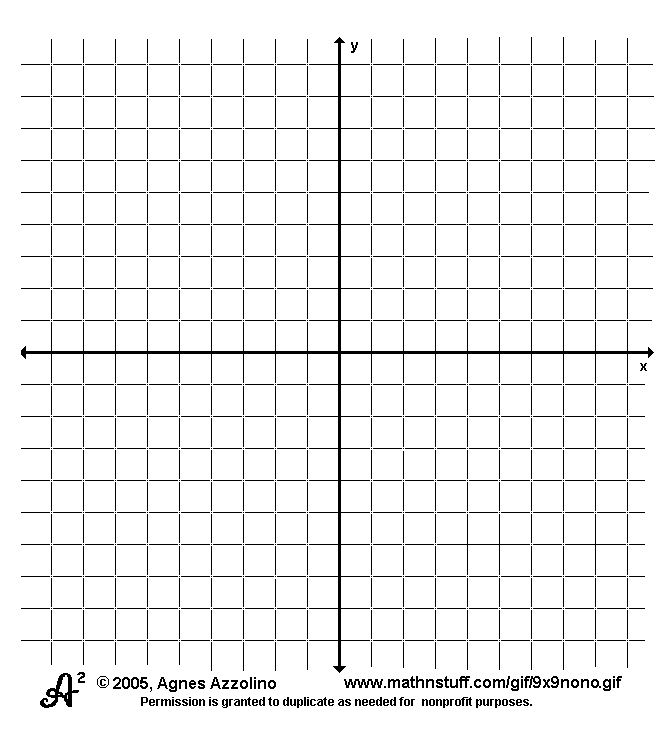
Corner point B uses the two inequalites http://vc.ws.edu/math1022/week7/linear2.gifand http://vc.ws.edu/math1022/week7/linear4.gifSo B is the solution to the system of equations http://vc.ws.edu/math1022/week7/linear11.gifand http://vc.ws.edu/math1022/week7/linear16.gif. This system can be solved by substitution.  The coordinates of point B are **(0,200).**

Corner point C uses the inequalities http://vc.ws.edu/math1022/week7/linear2.gifand http://vc.ws.edu/math1022/week7/linear3.gif. So, corner point C is the solution to the system of equations http://vc.ws.edu/math1022/week7/linear16.gifand http://vc.ws.edu/math1022/week7/linear17.gif. This system can be solved by either elimination or substitution (I will leave the math to you). The coordinates of corner point C are **(210 , 60)**.

Corner point D uses the inequalities http://vc.ws.edu/math1022/week7/linear5.gifand http://vc.ws.edu/math1022/week7/linear3.gif. Therefore, corner point D is the solution to the system of equations http://vc.ws.edu/math1022/week7/linear12.gifand http://vc.ws.edu/math1022/week7/linear17.gif. This can be solved by substitution and you will find the coordinates of point D are **(270,0).**

Last (hurray!!) - determine the corner point that maximizes our profit

Our profit equation is **40000x + 50000y.** We want to maximize this. We're going to select the corner point that maximizes this expression. To determine the desired corner point, we just plug the x and y coordinates into this expression and see which one gives us the biggest answer.



Point A (0, 0 ):   40000(0) + 50000(0) = 0 + 0 = 0 (I bet this won't be the one.)

Point B (0 , 200):   40000(0) + 50000(200) = 0 + 10,000,000 = 10,000,000

Point C (210 , 60):   40000(210) + 50000(60) = 8,400,000 + 3,000,000 = 11,400,000

Point D (270 , 0):   40000(270) + 50000(0) = 10,800,000 + 0 = 10,800,000

**Since Point C gave us the biggest profit, our decision would be to build 210 Carolina**

**homes and 60 Savannah homes. our profit would be $11,400,000.**

**This is our answer and we are done.**